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Observer Design for Lipschitz Nonlinear Systems A comparative study

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***Abstract*— In this paper the problem of nonlinear observer design for nonlinear systems is addressed. Based on recent results some methodologies are presented and the superiority or limitation of each one is discussed. The goal is to find the best less conservative approach in order to design a stable observer for a large class of nonlinear systems.**

***Keywords*— Lipshitz nonlinear system, nonlinear state observer, Lyapunov function, observer design, LMI, Ricatti equation..**

I. INTRODUCTION

In recent years there has been enormous activity in the study of different class of nonlinear systems. many researchers focus on Hybrid systems [1],[2]. Singularly perturbed systems has also a big interest , in fact most physical systems contain both slow and fast dynamics,[3]-[5]. Chaotic system, a nonlinear system which characterize by the sensitivity dependence on the initial conditions, has been a focal point of interest for many researchers [6],[7].The polynomial system is an important class of nonlinear systems which can describe the behaviour of many processes like electrical machines and robot manipulators,[8]-[10].

Researchers are still continuing to develop many approaches for the study and control of nonlinear systems. Therefore, the observer design for nonlinear systems is a subject of many research activities over the last three decades [11]-[14].This importance is due to the fact that the state estimation has various applications in many fields such as system monitoring, dynamic modelisation and fault detection.

The design of observers for nonlinear systems is a challenging problem,known the observers developed by Kalman and Luenberger for linear systems, different techniques have been proposed to deel with the nonlinearities. A first category of techniques knows as the extended Kalman and Luenberger observers which consists in considering the nonlinear system as linear around the estimated trajectory and applying linear algorithms. Second category is based on splitting the nonlinear system into a linear part and nonlinear one and choosing, then, the observer gain larger enough so that the linear part dominates the nonlinear one, these observers are known as high gain observers [15]. The third approach is based on transforming the nonlinear system into a linear one by a change of coordinates [16].

Indeed, many researches focus in a spatial class of nonlinear systems which is the class of Lipshitz systems, this class is very important ,in fact, any nonlinear system can be considered as a Lipshitz system at least locally. Observer design for Lipshitz systems was addressed for the first time by Thau[17] where a sufficient condition to ensure the stability of the observer was presented. After Thau, several researchers studied observer design for Lipshitz systems [18]. in which an algorithm to design an observer using the algebric Ricati equation is presented , this technique was extended in [19] in order to study identification and fault detection for Lipshitz nonlinear systems.

Later, many approaches are described in order to design an observer for Lipshitz systems. A fist category is based on solving the Linear Matrix Inequalities LMI [20],[21]. And the second category is based on the resolution of an algebric Ricati equation [22].

In this paper, these two techniques are presented based on many results published in order to do a comparative study.

The paper is organized as follow. Section II describes the system studied and the problem statement. Section III will presents some definitions and background results. Section IV is for describing the design of observer for our system. Section V illustrates the main difference between the studied approaches and finally section VI draws the conclusions.

II. STUDIED SYSTEM AND PROBLEM STATEMENT

Consider a nonlinear system represented by :

$$\begin{aligned} \dot{x} &= Ax + f(x,u) \\ y &= Cx \end{aligned} \qquad (1)$$

Where $x \in R^n$ is the state , $u \in R^p$ is the input, $y \in R^p$ is the output, the matrices $A \in R^{n*n}$,and $C \in R^{p*n}$ are such that the pair (A,C) is observable. The nonlinear function $f(x,u)$ is said to be locally Lipshitz in x uniformly with respect to u which satisfying the following condition :

There exists $k > 0$ such that

$$\| f(x_1,u) - f(x_2,u) \| \le k \| x_1 - x_2 \|, \ \forall x_1, x_2 \in \mathrm{X} \qquad (2)$$

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The observer will be considered to be of the form :

$$\dot{\hat{x}} = A\hat{x} + f(\hat{x},u) + L(y - C\hat{x}) \quad (3)$$

Where $\hat{x} \in R^n$ is the state vector of the observer. The matrix $L \in R^{n*p}$ represents the observer gain matrix to be calculated for state estimation.

Let $e = x - \hat{x}$ .then the dynamic of the estimation error is given by :

$$\dot{e} = (A - LC)e + f(x,u) - f(\hat{x},u), \forall x, \hat{x} \quad (4)$$

Now our objective is to formulate an observer gain L of (3) for the system (1) such that the error dynamics (4) is asymptotically stable

III. Definitions and background results

A. *The S-Procedure-Lemma:*

Let $V_0(x)$ and $V_1(x) \in R^n$,then $V_0(x) < 0 \quad \forall x \in R^n \setminus \{0\}$ satisfying $V_1(x) \le 0$ if and only if there exist $\varepsilon > 0$ such that

$$V_0(x) < \varepsilon V_1(x) \quad \forall x \in R^n \setminus \{0\} \quad (5)$$

B. *The Schur Inequality:*

The linear matrix inequality (LMI)

$$\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} < 0, Q = Q^T, R = R^T \quad (6)$$

Is equivalent to one of the following:

$$R < 0, Q - SR^{-1}S^T < 0 \quad (7)$$

Or:

$$Q < 0, R - S^T Q^{-1} S < 0 \quad (8)$$

IV. Observer synthesis

A. *Observer for Lipshitz nonlinear systems using LMI*

The main difficult thing in the design of state observers for nonlinear systems is to prove the convergence of the estimator error. we will show two different approaches proved by quadratic Lyapunov function and Lyapunov functional where the stability conditions can be expressed using LMIs.

Theorem1: [22]

Consider observer (3) for system (1) such as the condition (2) is verified ;then if there exist $\varepsilon > 0$,a gain matrix L and a symmetric positive definite matrix P such that :

$$\begin{pmatrix} (A - LC)^T + P(A - LC) + \varepsilon k^2 I & P \\ P & -\varepsilon I \end{pmatrix} < 0 \quad (9)$$

Then the estimation error converge exponentially to zero .

Proof of Theorem1:

Consider the Lyapunov function $V = e^T Pe$,where P is a symmetric positive definite matrix. Then $\dot{V} = \dot{e}^T Pe + e^T P\dot{e}$. From (1) and (3) ,we obtain

$$\begin{aligned} \dot{V} &= e^T[(A - LC)^T P + P(A - LC)]e \\ &+ e^T P[f(x,u) - f(\hat{x},u)] \\ &+ [f(x,u) - f(\hat{x},u)]^T Pe \end{aligned} \quad (10)$$

Converting (6) in matrix form we obtain:

$$\dot{V} = [e^T \quad \{f(x,u) - f(\hat{x},u)\}^T] \times \begin{pmatrix} (A - LC)^T P + P(A - LC) & P \\ P & 0 \end{pmatrix} \begin{bmatrix} e \\ f(x,u) - f(\hat{x},u) \end{bmatrix} \quad (11)$$

Of course, we should now proving that $\dot{V}$ is negative definite

From (2) we can have

$$\begin{aligned} &[f(x_1,u) - f(x_2,u)]^T [f(x_1,u) - f(x_2,u)] \\ &\le k^2 [x_1 - x_2]^T [x_1 - x_2] \end{aligned} \quad (12)$$

Hence after converting (12) in matrix form we obtain :

$$[(x - \hat{x})^T \quad \{f(x,u) - f(\hat{x},u)\}^T] \begin{pmatrix} -k^2 I & 0 \\ 0 & I \end{pmatrix} \begin{bmatrix} x - \hat{x} \\ f(x,u) - f(\hat{x},u) \end{bmatrix} \le 0 \quad (13)$$

Then

$$[(x - \hat{x})^T \quad \{f(x,u) - f(\hat{x},u)\}^T] M \begin{bmatrix} x - \hat{x} \\ f(x,u) - f(\hat{x},u) \end{bmatrix} \le 0 \quad (14)$$

Where $$M = \begin{pmatrix} -k^2 I & 0 \\ 0 & I \end{pmatrix} \quad (15)$$

Applying the S-Procedure Lemma to (10) and (14) we find that $\dot{V} \le 0$ if and only if there exists ε verifying that

$$\begin{pmatrix} (A - LC)^T P + P(A - LC) & P \\ P & 0 \end{pmatrix} - \varepsilon M < 0 \quad (16)$$

This leads to (8)

And after the change of variable $L = P^{-1}Y$:

$$\begin{pmatrix} (A^T P - C^T Y^T + PA - YC + \varepsilon k^2 I & P \\ P & -\varepsilon I \end{pmatrix} < 0 \quad (17)$$

Theorem2: [21]

Consider observer (3) for system (1) such that f verifying (2) ,then if there exist β , c ,a symmetric definitive positive matrices P and R and a gain matrix L such that

$$\frac{P}{k^2} > \frac{R}{\beta} + \frac{c}{\beta} I \quad (18)$$

$$\begin{pmatrix} (A - LC)^T + P(A - LC) + \beta P & P \\ P & -cI \end{pmatrix} < 0 \quad (19)$$

Then the estimation error converges asymptotically to zero.

Proof of Theorem2:

Consider the Lyapunov functional:

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$$V = e^T Pe + \lambda_{\min}(Q)e^{-\beta t}\int \left\| f(x(t),u) - f(\hat{x}(t),u)\right\|^2 dt \quad (19)$$

Then

$$\begin{aligned}\dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\ &-\beta\lambda_{\min}(Q)\int \left\| f(x(t),u) - f(\hat{x}(t),u)\right\|^2 dt \\ &+\lambda_{\min}(Q)e^{-\beta t}\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (20)$$

From (4) we obtain :

$$\begin{aligned}\dot{V} &= e^T[(A-LC)^T P + P(A-LC)]e \\ &+e^T P\left| f(x,u) - f(\hat{x},u)\right| \\ &+\left| f(x,u) - f(\hat{x},u)\right|^T Pe \\ &-\beta\lambda_{\min}(Q)e^{-\beta t}\int \left\| f(x(t),u) - f(\hat{x}(t),u)\right\|^2 dt \\ &+\lambda_{\min}(Q)e^{-\beta t}\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (21)$$

And from (19) :

$$\begin{aligned}\dot{V} &= e^T[(A-LC)^T P + P(A-LC)]e \\ &+e^T P\left| f(x,u) - f(\hat{x},u)\right| \\ &+\left| f(x,u) - f(\hat{x},u)\right|^T Pe \\ &-\beta V + \beta e^T Pe + \lambda_{\min}(Q)e^{-\beta t}\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (22)$$

We have $e^{-\beta t} \le 1 \ \forall t \ge 0$ then:

$$\begin{aligned}\dot{V} &\le e^T[(A-LC)^T P + P(A-LC)]e \\ &+e^T P\left| f(x,u) - f(\hat{x},u)\right| \\ &+\left| f(x,u) - f(\hat{x},u)\right|^T Pe \\ &-\beta V + \beta e^T Pe + \lambda_{\min}(Q)\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (23)$$

As $V \ge e^T Pe$ and using (17) we have

$$V \ge e^T Pe \ge \frac{k^2}{\beta}(e^T Qe + c\left\|e\right\|^2 \quad (24)$$

(2) provides $$k \ge \frac{\left\| f(x,u) - f(\hat{x},u)\right\|}{\left\|e\right\|}, e \ne 0 \quad (25)$$

So from (24) we obtain

$$\begin{aligned}V &\ge e^T Pe \ge \frac{1}{\beta}(\left\| f(x,u) - f(\hat{x},u)\right\|^2 \frac{e^T}{\left\|e\right\|} Q \frac{e}{\left\|e\right\|} \\ &+c\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (26)$$

And as $\lambda_{\min}(Q)\left\|e\right\| \le e^T Qe, \forall e \in R^n$

From (26)

$$\begin{aligned}V &\ge e^T Pe \ge \frac{1}{\beta}(\left\| f(x,u) - f(\hat{x},u)\right\|^2 \frac{e^T}{\left\|e\right\|} Q \frac{e}{\left\|e\right\|} \\ &+c\left\| f(x,u) - f(\hat{x},u)\right\|^2 \\ &\ge \frac{1}{\beta}(\lambda_{\min}(Q) + c)\left\| f(x,u) - f(\hat{x},u)\right\|^2\end{aligned} \quad (27)$$

After multiplying for $-\beta$ we obtain from (27)

$$\lambda_{\min}(Q)\left\| f(x,u) - f(\hat{x},u)\right\|^2 - \beta V \le -c\left\| f(x,u) - f(\hat{x},u)\right\|^2 \quad (28)$$

Using (23) we have

$$\begin{aligned}\dot{V} &\le e^T[(A-LC)^T P + P(A-LC) + \beta P]e \\ &+e^T P\left| f(x,u) - f(\hat{x},u)\right| + \left| f(x,u) - f(\hat{x},u)\right|^T Pe \\ &-c\left| f(x,u) - f(\hat{x},u)\right|^T \left| f(x,u) - f(\hat{x},u)\right|\end{aligned} \quad (29)$$

Therefore $\dot{V}$ is definite negative if (17) is satisfied.

(17) and (18) are an LMI and we obtain for a fixed β after the change of variable $L = P^{-1}Y$:

$$\begin{pmatrix} A^T P - C^T Y^T + PA - YC + \beta P & P \\ P & -cI \end{pmatrix} < 0 \quad (30)$$

Condition (17) and (18) together with (30) are LMIs involving two unknown matrices P and Y .it is easy also to verify that (18) and (19) imply (9) ,however the converse is not true.

B. Observer design using Ricatti Equations

Theorem3: [22]

There exist an observer (3) for system (1) such that (4) is quadraticlly stabilized if and only if there exist $\varepsilon > 0$ and $\alpha \in R$ such that the following Ricatti inequality has a symmetric definite positive solution P :

$$A^T P + PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP - \alpha^2 C^T C < 0 \quad (31)$$

The observer gain L will be chosen as

$$L = \frac{\alpha^2}{2}P^{-1}C^T \quad (32)$$

Proof of Theorem3:

Review to (9) and by applying the Schur Lemma (7) we obtain

$$(A-LC)^T P + P(A-LC) + \varepsilon k^2 I + \frac{1}{\varepsilon}PP < 0 \quad (33)$$

So:

$$A^T P + +PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP - C^T L^T P - PLC < 0 \quad (34)$$

Then we can have

$$e^T(A^T P + PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP)e < 0 \quad (35)$$

There exist so $\alpha \in R$ such that

$$e^T(A^T P + PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP)e - \alpha^2 e^T C^T Ce < 0 \quad \forall e \quad (36)$$

Equation (31) is then a necessary condition.

Equation (31) can be written as

$$A^T P + PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP - \alpha^2 C^T C + \mu I < 0 \quad (37)$$

For given ε , k , α and μ we can easily solve the following Ricatti equation :

$$A^T P + PA + \varepsilon k^2 I + \frac{1}{\varepsilon}PP - \alpha^2 C^T C + \mu I = 0 \quad (38)$$

It may be solved using the MATLAB command “ARE”.

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We can easily notice that the Ricatti equation has only one unknown matrix P ,so it is easier to solve this equation than to solve a LMI inequality.

V. ILLUSTRATIVE EXAMPLE

A. Dynamic model for a single link robot

Consider the model of a single link robot :

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{J_1} + \frac{k}{J_1}x_3 - \frac{mgl}{J_1}\sin x_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k}{J_2}x_1 - \frac{k}{J_2}x_3 + \frac{l}{J_2}u \end{aligned} \tag{39}$$

Where x_1 and x_2 are the link displacement and its velocity,respectively, x_3 and x_4 are the rotor displacement and its velocity,respectively. J_1, J_2, k, l and g are the link inertia,the rotor inertia,the elastic constant,the position of the center of mass and the gravity acceleration,respectively.

We can put this model in the form of :

$$\begin{aligned} \dot{x} &= Ax + Bu + f(x) \\ y &= Cx \end{aligned} \tag{40}$$

With $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -48.6 & 0 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 48.6 & 0 & 48.6 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 26.6 \end{pmatrix} \tag{41}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad f(x) = \begin{pmatrix} 0 \\ -3.33\sin x1 \\ 0 \\ 0 \end{pmatrix}$$

The Lipschitz constant for this system is equal to 3.33.

B. Observer design for a single link robot

Refer to Theorem 1 and after solving the LMI (17) with $\varepsilon = 1$ we obtain :

$$P = \begin{pmatrix} 0.7455 & 0 & 0 & 0 \\ 0 & 0.8064 & 0.1588 & 0.3665 \\ 0 & 0.1588 & 0.7937 & 0.0718 \\ 0 & 0.3665 & 0.0718 & 0.3960 \end{pmatrix} \tag{42}$$

$$L = \begin{pmatrix} 5.7499 & 11.4371 \\ 28.2831 & 16.5140 \\ 2.4049 & -29.5317 \\ -30.2335 & -12.0615 \end{pmatrix} \tag{43}$$

The eigenvalues of $(A - LC)$ are :

$$eig(A - LC) = \begin{pmatrix} -56.5023 \\ -38.3702 \\ -4.3677 \\ -0.2359 \end{pmatrix} \tag{44}$$

The eigenvalues of $(A - LC)$ are negatives so the system is stable. Also the eigenvalues are well damped ,the response will show no transient oscillations. However, there is two small eigenvalues. So the estimated state will converge slowly to the actual state.

Using Theorem 3 with solving the Ricatti equation with $\varepsilon = 1$ and $\alpha = 180$ we obtain :

$$P = \begin{pmatrix} 1.2812 & 0 & 0 & 0 \\ 0 & 1.2812 & 0.0038 & 0.0028 \\ 0 & 0.0038 & 1.9165 & 0.0941 \\ 0 & 0.0028 & 0.0941 & 1.7473 \end{pmatrix} \tag{45}$$

$$L = \begin{pmatrix} 2.1985 & -10.1764 \\ 9.2095 & 2.1985 \\ -0.017 & -0.0480 \\ -0.0469 & 0.0075 \end{pmatrix} \tag{46}$$

The eigenvalues of $(A - LC)$ are :

$$eig(A - LC) = \begin{pmatrix} -2.1985 + 10.1637i \\ -2.1985 - 10.1637i \\ + 0.1818i \\ - 0.1818i \end{pmatrix} \tag{47}$$

In this case eigenvalues have lower damping and the overall eigenvalues are large. So the estimated states rapidly converge to the actual state and transient oscillations are not present. The transient performance is better than the previous LMI solution.

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VI. Comparison between the presented approaches

In the first approach, the convergence of the estimation error has been studied with quadratic Lyapunov function and Lyapunov functional and the condition of stability has been expressed using LMI with two unknown variables , which are difficult to be satisfied when the Lipshitz constant is large.
For the second approach ,it requires an algebric Ricatti equation to be solved with one variable which yields for a stable observer for larger Lipshitz constants and that make the procedure implement is easy. The important thing to show the superiority of the second approach is that it is less conservative

VII. Conclusions

We have addressed the design problems of observer for a class of Lipshitz nonlinear systems .results show the advantage of the resolution of the Ricatti equation than the standard LMI design technique. So as the subject of future investigations, this approach may be used to design observers for more complex systems like for example polynomial systems.

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